Spring 2015

1.

Since the €24 includes an 18% tip and 10% tax it is 1.28 times the original price, making the original price $\frac{24}{1.28} = 18.75$, then we multiply by 1.2 to get the price in dollars.

$18.75 * 1.2 = 22.50$

So the answer is **B. 22.5**

2.

$a_n = a_1 + (n-1)d$

$a_n = 47$

$a_1 = 2$

So $(n-1)d = 45$

$d = 3$ $(lowest\ factor\ of\ 45\ greater\ than\ 1)$

So $n - 1 = 15, n = 16$

So the answer is **C. 16**

3.

Just plug in $(2, -4)$ for (x, y)

$3(2) + (-4) = a$

$(2) - 2(-4) = b$

$a + b = 9 + 6$

So the answer is **D. 12**

4.

Since the average of Al's five is 20, his total credits are 100.

Since the average of all six is 24, the total is 144. The one Al doesn't have is 44

So the answer is **C. 44**

5.

$a_2 = 2a_1 - 1 \; so \; it \; has \; to \; be \; an \; odd)$
$a_3 = \frac{a_2}{3} (\; so \; a_2 \; is \; divisible \; by \; 3)$
$a_2 \; must \; be \; 3 \; or \; 9$
$a_3 \; must \; be \; 1 \; or \; 3$
$a_4 = 12 - 2(a_3)$
$a_4 = \; 10 \; or \; 6$
$a_4 \neq 10 \; (single \; digit \; integers \; only)$

So the answer is **D. 6**

6. Let a_1 denote the knight. Since he is telling the truth, we can say without loss of generality that a_2 must be a knight and a_9 a knave. Then since a_2 is telling the truth a_3 must be a knave. The only way for a_3 to be lying is for $a_4$to be a knight, making a_5 a knight, and a_6 a knave, finally making a_7 and a_8 knights, for a total of 3 knaves.

So the answer is **C. 3**

7.

$\frac{A \; (even)}{MA \; (mult.of \; 8)}$=.TYC $(Has \; to \; be \; .125, \; .375, .625, .875)$

$\frac{2}{32} = .0625 \; (exclude)$

$\frac{4}{24} = .166 \; (exlude)$

$\frac{6}{16} = .375$

6+3=9

So the answer is **E. 9**

8.

Using synthetic division:

$$\begin{array}{r|rrrrrr} 1 & 2 & -4 & 1 & a & b & c \\ & & 2 & -2 & -1 & a-1 & b+a-1 \\ \hline & 2 & -2 & -1 & a-1 & b+a-1 & c+b+a-1 \end{array}$$

$c + b + a = 1$

$1|2 \;\; -2 \;\; -1 \;\; a-1 \;\; b+a-1$

$$\begin{array}{cccccc} & 2 & & & \\ \hline 2 & 0 & -1 & a-2 & b+2a-3 \end{array}$$

$b + 2a = 3$
$c + b + a = 1$
$-c - b - a = -1$
$a - c = 2$
So the answer is **A. 2**

9.

Xy-6x+4y=36

(x+4)y+6x+36

$y = \frac{6(x+6)}{x+4}$

So there is an asymptote at $x = -4$

(y-6)x=-4y+36

$x = -\frac{4(y-9)}{y-6}$

So there is an asymptote at $y = 6$

It must be symmetric at the asymptotes if at all, so

xy=-24

So the answer is **B. -24**

10.

Find a_4

11.

$a^3 + b^3 + c^2 = 2015$

$c = \sqrt{2015 - a^3 - b^3}$

Using a calculator, starting with $y = \sqrt{2015 - x^3 - 2^3}$ and incrementing the 2 as needed, we get:

(a, b, c) =(11,2,26)

2+11+26=39

So the answer is **B. 39**

12.

$f(0) = kf(1)\ (plug\ 0\ for\ x)$

$f(1) = kf(0)\ (plug\ 1\ for\ x)$

$k = \frac{1}{k}$

$k =$ 1, -1, but must be greater than 0

So the answer is **D. 1**

13 Our tripled numbers must be multiples of 3 and be between 3000 and 6045, with all odd digits.

3111, 3117, 3171, 3177, 3711, 3717, 3771, and 3777 make 8.

3333, 3999 work similarly to make 8 more

3555 makes 1 more

3357 and its rotations make 6 more

3159 and its rotations make 6 more

Same with 3759, and 3153, and rotations for 12 more

5133, 5199, 5313, etc make 12 more

Same with 5733, 5739, etc for 12 more

5553, 5535, 5355, 5559, 5595, 5955, for 6 more

5511, 5577, etc for 12 more

Total is 83

So the answer is **D. 83**

14.

Take the count for the 6^{th} hour and divide it by 2; add 256. That will give 5^{th} hour count. Work backwards from this until 1^{st} hour count is reached.

6^{th} hour – 256 bacteria

5^{th} hour – 384 bacteria

4^{th} hour – 448 bacteria

3^{rd} hour – 489 bacteria

2^{nd} hour – 496 bacteria

1^{st} hour – 504 bacteria

So the answer is **504**

15. $2\sqrt{y} - x + 1 = 0$

$x = 2\sqrt{y} + 1$

$2\sqrt{y} + 1 + y = 25$

$y + 2\sqrt{y} + 1 = 25$

$\left(\sqrt{y} + 1\right)^2 = 25$

$\sqrt{y} + 1 = \pm 5$

$\sqrt{y} = -1 \pm 5 = 4, -6$

$y = 16\ (since \sqrt{y}\ cannot\ be\ negative)$

$x = 9$

16-9=7

So the answer is **A. 7**

16 There are $4 * 3 * 4 * 4 * 3 * 4$ ways he can travel, and there is a $(\frac{1}{4} + \frac{1}{3} + \frac{1}{4} - \frac{1}{12} - \frac{1}{12} - \frac{1}{16} + \frac{1}{48})$ chance that any particular way has the same method between two matching legs.

Multiplying$4^4 * 3^2 \left(\frac{12+16+12-4-4-3+1}{48}\right) = 48(30) = 1440$

So the answer is **C. 1440**

17 The sum of the first n integers is $\frac{n}{2}(n+1)$, and this has to equal the sum of 5 integers starting at a ($\frac{5}{2}(a+a+4)=5a+10$) and the 8 integers starting at b ($\frac{8}{2}(b+b+7)=8b+28$), so $\frac{n}{2}(n+1)$ needs to be a multiple of 5 and a multiple of 4 but not 8, meaning $n(n+1)$ needs to be a multiple of 8 but not 16. Looking at the multiples of 8, 8 doesn't have a multiple of 5 next to it, but 24 does. So our sum can be $\frac{24}{2}(25)=300$.

Then $5a+10=300, so\ 5a=290, a=58$

And $8b+28=300, 8b=272, b=34$

58-34=24

So the answer is **C. 24**

18 The shortest chain to 0 I could find was

$\frac{7}{9} \to -\frac{9}{7} \to -\frac{2}{7} \to \frac{5}{7} \to -\frac{7}{5} \to -\frac{2}{5} \to \frac{3}{5} \to -\frac{5}{3} \to -\frac{2}{3} \to \frac{1}{3} \to -3 \to -2 \to -1 \to 0$

Which is 13 steps

So the answer is **E. 13**

19 In order to make two isosceles triangles, the diagonal must match one of the given sides. If it matches the 4, Then the diagonal, the 4 side, and the 6 side make one triangle (since 4, 4, 10 can't make a triangle), and the missing side must be a 10 to make the other triangle isosceles. This gives an area of $3\sqrt{16-9}+2\sqrt{100-4}=3\sqrt{7}+8\sqrt{6}\approx 27.5$

If the diagonal is 6, then we must have a 6,4,6 and a 10,6,10, since orienting the other way can't get another isosceles triangle. This gives an area of
$2\sqrt{36-16}+3\sqrt{100-9}=4\sqrt{5}+3\sqrt{91}\approx 37.6$

If the diagonal is 10, then we have a 10, 4, 10 and a 6, 10, 6. This has an area of
$2\sqrt{100-4}+5\sqrt{36-25}=8\sqrt{6}+5\sqrt{11}\approx 36.2$

$36.2-27.5=8.7$

So the answer is **D. (8…9)**

20.For each value we can double it, halve it, get rid of it, or keep it the same. Let's make a table of how this would change our sum:

Delete	-8	-12	-22	-24	-29

Half	-4	-6	-11	-12	-14.5
Same	0	0	0	0	0
Double	+8	+12	+22	+24	+29

We need to delete exactly one and be able to make up for it. If we delete the 29, there isn't a way to get a net gain of 29 from the other values.

If we delete the 24, we can add 22 and 8 and subtract 6 to get a gain of 24 so we're back to the same sum.

So the answer is **D. 24**